1 Introduction

The formulae shown here are for use where the end points or distances are less than, or equal to, 1500km.

2 The Direct Problem: Robbin's Formulae

Given: $\phi_1, \lambda_1, \alpha_{12}, s$.

Find: $\phi_2, \lambda_2, \alpha_{21}$.

Then:

$$
\begin{aligned}
h &= e^{'} \cos\phi_1 \cos\alpha_{12} && (1) \\
g &= e^{'} \sin\phi_1 && (2) \\
\eta &= s/\nu_1 && (3) \\
\sigma &= \eta[1+\eta^2 \frac{h^2(1-h^2)}{6} - \eta^3 \frac{gh(1-2h^2)}{8} \\
&\quad -\eta^4(\frac{h^2(4-17h^2)}{120} - 3g^2(1-7h^2)) + \eta^5 \frac{gh}{48}] && (4) \\
\sin\zeta_2 &= \sin\phi_1 \cos\sigma + \cos\phi_1 \cos\alpha_{12} \sin\sigma && (5) \\
\sin\Delta\lambda &= \frac{\sin\sigma \sin\alpha_{12}}{\cos\zeta_2} && (6) \\
\sin\alpha_{21}^{'} &= \frac{-\cos\phi_1 \sin\alpha_{12}}{\cos\zeta_2} && (7) \\
& && (8)
\end{aligned}
$$

If ζ_2, $\Delta\lambda$ or $\alpha_{21}^{'} \rightarrow \frac{\pi}{2}$ then:

$$
\begin{aligned}
\cot\Delta\lambda &= \frac{\cos\phi_1 \cot\sigma - \sin\phi_1 \cos\alpha_{12}}{\sin\alpha_{12}} && (9) \\
\tan\zeta_2 &= \frac{\sin\phi_1 \Delta\lambda + \sin\Delta\lambda \cot\alpha_{12}}{\cos\phi_1} && (10) \\
\tan\alpha_{21}^{'} &= \frac{\sin\alpha_{12}}{\cos\sigma \cos\alpha_{12} - \sin\sigma \tan\phi_1} && (11)
\end{aligned}
$$

$$
\begin{aligned}
\mu &= 1 + \frac{e^{'2}(\sin\zeta_2 - \sin\phi_1)^2}{2} && (12) \\
\tan\phi_2 &= (1+e^{'2})(1 - \frac{e^2 \mu \sin\phi_1}{\sin\zeta_2} \tan\zeta_2) && (13) \\
\lambda_2 &= \lambda_1 + \Delta\lambda && (14) \\
\alpha_{21} &= \alpha_{21}^{'} - (\phi_2 - \zeta_2) \sin\alpha_{21}^{'} \tan\frac{\sigma}{2} && (15)
\end{aligned}
$$

3 The Reverse Problem: Robbin's Formulae

Given: ϕ_1, λ_1, ϕ_2, λ_2.

Find: α_{12}, α_{21}, s.

Then:

$$\begin{aligned}
\tan\zeta_2 &= (1-e^2)\tan\phi_2 + \frac{e^2\nu_1\sin\phi_1}{\nu_2\cos\phi_2} && (16)\\
\tau_1 &= \cos\phi_1\tan\zeta_2 - \sin\phi_1\cos(\lambda_2-\lambda_1) && (17)\\
\tan\alpha_{12} &= \frac{\sin(\lambda_2-\lambda_1)}{\tau_1} && (18)\\
\tan\zeta_1 &= (1-e^2)\tan\phi_1 + \frac{e^2\nu_2\sin\phi_1}{\nu_1\cos\phi_1} && (19)\\
\tau_2 &= \cos\phi_2\tan\zeta_1 - \sin\phi_2\cos(\lambda_1-\lambda_2) && (20)\\
\tan\alpha_{21} &= \frac{\sin(\lambda_1-\lambda_2)}{\tau_2} && (21)\\
\chi &= \frac{\sin(\lambda_2-\lambda_1)}{\sin\alpha_{12}} && (22)
\end{aligned}$$

If $\sin\alpha_{12} \to 0$ then:

$$\begin{aligned}
\chi &= \frac{\tau_1}{\cos\alpha_{12}} && (23)\\
\sin\sigma &= \chi\cos\zeta_2 && (24)\\
g &= e^{'}\sin\phi_1 && (25)\\
h &= e^{'}\cos\phi_1\cos\alpha_{12} && (26)\\
s &= \nu_1\sigma[1-\sigma^2\frac{h^2(1-h^2)}{6}+\sigma^3\frac{gh(1-2h^2)}{8} \\
&\quad +\sigma^4\frac{h^2(4-7h^2)-3g^2(1-7h^2)}{120}-\sigma^5\frac{gh}{48}] && (27)
\end{aligned}$$

4 Definitions

a: Semi Major Axis.
b: Semi Minor Axis.

$$\begin{aligned}
e^2 &= \frac{a^2-b^2}{a^2} && (28)\\
e^{'2} &= \frac{a^2-b^2}{b^2} && (29)\\
\nu_n &= \sqrt{\frac{a}{1-e^2\sin^2\phi_n}} && (30)
\end{aligned}$$

`$Id$`