

Co-ordinate transformations of this type do not maintain shape across the procedure. There are three independent scale factors.

Transforming  $(x, y, z)$  into  $(X, Y, Z)$  by:

$\omega$  = Rotation around the  $x$  axis.

$\phi$  = Rotation around the  $y$  axis.

$\kappa$  = Rotation around the  $z$  axis.

$h$  = Scaling in  $x$ .

$k$  = Scaling in  $y$ .

$n$  = Scaling in  $z$ .

$X_S$  = Shift in  $X$ .

$Y_S$  = Shift in  $Y$ .

$Z_S$  = Shift in  $Z$ .

$x_S$  = Shift in  $x$ .

$y_S$  = Shift in  $y$ .

$z_S$  = Shift in  $z$ .

Then:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R \begin{bmatrix} hx \\ ky \\ nz \end{bmatrix} + \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} \quad (1)$$

$$= R \begin{bmatrix} h(x+x_S) \\ k(y+y_S) \\ (nz+z_S) \end{bmatrix} \quad (2)$$

Where  $R$ , the rotation matrix, is defined to be:

$$R = \begin{bmatrix} \cos \phi \cos \kappa & -\cos \phi \sin \kappa & \sin \phi \\ \sin \omega \sin \phi \cos \kappa + \cos \omega \sin \kappa & -\sin \omega \sin \phi \sin \kappa + \cos \omega \cos \kappa & -\sin \omega \cos \kappa \\ -\cos \omega \sin \phi \cos \kappa + \sin \omega \sin \kappa & \cos \omega \sin \phi \sin \kappa + \sin \omega \cos \kappa & \cos \omega \cos \phi \end{bmatrix} \quad (3)$$

$$R^T = \begin{bmatrix} \cos \phi \cos \kappa & \sin \omega \sin \phi \cos \kappa + \cos \omega \sin \kappa & -\cos \omega \sin \phi \cos \kappa + \sin \omega \sin \kappa \\ -\cos \phi \sin \kappa & -\sin \omega \sin \phi \sin \kappa + \cos \omega \cos \kappa & -\sin \omega \sin \phi \sin \kappa + \cos \omega \cos \kappa \\ \sin \phi & -\sin \omega \cos \kappa & \cos \omega \cos \phi \end{bmatrix} \quad (4)$$

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