

Co-ordinate transformations of this type maintain shape across the procedure. There is a single scale factor.

Transforming (x, y, z) into (X, Y, Z) by:

ω = Rotation around the x axis.

ϕ = Rotation around the y axis.

κ = Rotation around the z axis.

k = Scaling.

X_S = Shift in X .

Y_S = Shift in Y .

Z_S = Shift in Z .

x_S = Shift in x .

y_S = Shift in y .

z_S = Shift in z .

Then:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = kR \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} \quad (1)$$

$$= kR \begin{bmatrix} x+x_S \\ y+y_S \\ z+z_S \end{bmatrix} \quad (2)$$

Where R , the rotation matrix, is defined to be:

$$R = \begin{bmatrix} \cos \phi \cos \kappa & -\cos \phi \sin \kappa & \sin \phi \\ \sin \omega \sin \phi \cos \kappa + \cos \omega \sin \kappa & -\sin \omega \sin \phi \sin \kappa + \cos \omega \cos \kappa & -\sin \omega \cos \kappa \\ -\cos \omega \sin \phi \cos \kappa + \sin \omega \sin \kappa & \cos \omega \sin \phi \sin \kappa + \sin \omega \cos \kappa & \cos \omega \cos \phi \end{bmatrix} \quad (3)$$

$$R^T = \begin{bmatrix} \cos \phi \cos \kappa & \sin \omega \sin \phi \cos \kappa + \cos \omega \sin \kappa & -\cos \omega \sin \phi \cos \kappa + \sin \omega \sin \kappa \\ -\cos \phi \sin \kappa & -\sin \omega \sin \phi \sin \kappa + \cos \omega \cos \kappa & -\sin \omega \sin \phi \sin \kappa + \cos \omega \cos \kappa \\ \sin \phi & -\sin \omega \cos \kappa & \cos \omega \cos \phi \end{bmatrix} \quad (4)$$

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